Smooth energy

Given a manifold M and a vector field X on M, where all integral lines of X are geodesics. The smoothness of a polygon $\gamma \subset M$ is:

$$E_X(\gamma) := \int_\gamma \langle \dot{\gamma}, X \rangle^2 \, ds.$$

In the discrete setting:

$$E(\gamma) = \sum_{\text{edges } e} \langle e, X \rangle^2 .$$

and the variation of a vertex $v \in \gamma$ with local coordinates (v_x, v_y) w.r.t. the basis frame $(-JX, X)$ is:

$$\begin{aligned}
\delta_{v_y} E(\gamma) &= \sum_{\text{edges } e} 2 \left\langle \delta_{v_y} e, X \right\rangle \\
&= 2 \left[\langle e_-, X \rangle \, \delta_{v_y} e_- F + \langle e_+, X \rangle \, \delta_{v_y} e_+ F \right] \\
&= 2 \langle e_- - e_+, F \rangle \, \delta_{v_y} e_- F = 2 \langle e_- - e_+, F \rangle \, F.
\end{aligned}$$